

Groups and Graphs, Designs and Dynamics

Edited by

R.A. BAILEY

University of St Andrews, Scotland

PETER J. CAMERON

University of St Andrews, Scotland

YAOKUN WU

Shanghai Jiao Tong University, China

Contents

<i>List of authors</i>	<i>page</i> xi
<i>Preface</i>	xiii
1 Topics in representation theory of finite groups <i>T. Ceccherini-Silberstein, F. Scarabotti and F. Tolli</i>	1
1.1 Introduction	2
1.2 Representation theory and harmonic analysis on finite groups	4
1.2.1 Representations	5
1.2.2 Finite Gelfand pairs	17
1.2.3 Spherical functions	21
1.2.4 Harmonic analysis of finite Gelfand pairs	25
1.3 Laplace operators and spectra of random walks on finite graphs	26
1.3.1 Finite graphs and their spectra	27
1.3.2 Strongly regular graphs	38
1.4 Association schemes	41
1.5 Applications of Gelfand pairs to probability	46
1.5.1 Markov chains	46
1.5.2 The Ehrenfest diffusion model	52
1.6 Induced representations and Mackey theory	56
1.6.1 Induced representations	56
1.6.2 Mackey theory	59
1.6.3 The little group method of Mackey and Wigner	61
1.6.4 Hecke algebras	63
1.6.5 Multiplicity-free triples and their spherical functions	66
1.7 Representation theory of $GL(2, \mathbb{F}_q)$	69
1.7.1 Finite fields and their characters	69

1.7.2	Representation theory of the affine group $\text{Aff}(\mathbb{F}_q)$	73
1.7.3	The general linear group $\text{GL}(2, \mathbb{F}_q)$	75
1.7.4	Representations of $\text{GL}(2, \mathbb{F}_q)$	77
	<i>References</i>	82
2	Quantum probability approach to spectral analysis of growing graphs <i>N. Obata</i>	87
2.1	Introduction	87
2.2	Basic concepts of quantum probability	92
2.2.1	Algebraic probability spaces	92
2.2.2	Spectral distributions	94
2.2.3	Convergence of random variables	97
2.2.4	Classical probability vs quantum probability	98
2.2.5	Notes	100
2.3	Quantum decomposition	100
2.3.1	Jacobi coefficients and interacting Fock spaces	100
2.3.2	Orthogonal polynomials	102
2.3.3	Quantum decomposition	106
2.3.4	How to explicitly compute μ from $(\{\omega_n\}, \{\alpha_n\})$	109
2.3.5	Boson, fermion and free Fock spaces	114
2.3.6	Notes	118
2.4	Spectral distributions of graphs	118
2.4.1	Adjacency matrix as a real random variable	118
2.4.2	IFS structure associated to graphs	120
2.4.3	Homogeneous trees and Kesten distributions	125
2.5	Growing graphs	130
2.5.1	Formulation of question in general	130
2.5.2	Growing distance-regular graphs	133
2.5.3	Growing regular graphs	139
2.5.4	Notes	142
2.6	Concepts of independence and graph products	143
2.6.1	From classical to commutative independence	143
2.6.2	Graph products	148
2.6.3	Central Limit Theorem for Cartesian powers	152
2.6.4	Monotone independence and comb product	154
2.6.5	Boolean independence and star product	159
2.6.6	Convolutions of spectral distributions	161
2.6.7	Notes	171
	<i>References</i>	172

3	Laplacian eigenvalues and optimality	<i>R. A. Bailey and P. J. Cameron</i>	176
3.1	Block designs in experiments		176
3.1.1	Experiments in blocks		177
3.1.2	Complete-block designs		181
3.1.3	Incomplete-block designs		183
3.1.4	Matrix formulae		186
3.1.5	Eigenspaces of real symmetric matrices		191
3.1.6	Fisher's Inequality		192
3.1.7	Constructions		193
3.1.8	Partially balanced designs		195
3.1.9	Laplacian matrix and information matrix		197
3.1.10	Estimation and variance		198
3.1.11	Reparametrization		200
3.1.12	Exercises		201
3.2	Laplacian matrices and their eigenvalues		202
3.2.1	Which graph is best?		203
3.2.2	Graph terminology		204
3.2.3	The Laplacian of a graph		207
3.2.4	Isoperimetric number		210
3.2.5	Signed incidence matrix		212
3.2.6	Generalized inverse; Moore–Penrose inverse		212
3.2.7	Electrical networks		214
3.2.8	The Matrix-Tree Theorem		218
3.2.9	Markov chains		222
3.2.10	Exercises		224
3.3	Designs, graphs and optimality		225
3.3.1	Two graphs associated with a block design		225
3.3.2	Laplacian matrices		228
3.3.3	Estimation and variance		230
3.3.4	Resistance distance		230
3.3.5	Spanning trees		234
3.3.6	Measures of optimality		236
3.3.7	Some optimal designs		238
3.3.8	Designs with very low replication		242
3.3.9	Exercises		246
3.4	Further topics		246
3.4.1	Sylvester designs		247
3.4.2	Sparse versus dense		250
3.4.3	Variance-balanced designs		251

3.4.4	Recognising a concurrence graph	255
3.4.5	Other graph parameters	256
3.4.6	Some open problems	260
3.4.7	Exercises	262
	<i>References</i>	264
4	Symbolic dynamics and the stable algebra of matrices <i>M. Boyle and S. Schmieding</i>	266
4.1	Overview	267
4.2	Basics	271
4.2.1	Topological dynamics	271
4.2.2	Symbolic dynamics	272
4.2.3	Edge SFTs	273
4.2.4	The continuous shift-commuting maps	274
4.2.5	Powers of an edge SFT	276
4.2.6	Periodic points and nonzero spectrum	276
4.2.7	Classification of SFTs	277
4.2.8	Strong shift equivalence of matrices, classification of SFTs	278
4.2.9	Shift equivalence	280
4.2.10	Williams' shift equivalence conjecture	280
4.2.11	Notes	282
4.3	Shift equivalence and strong shift equivalence over a ring	292
4.3.1	$\text{SE-}\mathbb{Z}_+$: dynamical meaning and reduction to $\text{SE-}\mathbb{Z}$	292
4.3.2	Strong shift equivalence of matrices over a ring	292
4.3.3	SE, SSE and $\det(I - tA)$	294
4.3.4	Shift equivalence over a ring $\mathcal{R}$	294
4.3.5	$\text{SIM-}\mathbb{Z}$ and $\text{SE-}\mathbb{Z}$: some example classes	298
4.3.6	$\text{SE-}\mathbb{Z}$ via direct limits	299
4.3.7	$\text{SE-}\mathbb{Z}$ via polynomials	301
4.3.8	Cokernel of $(I - tA)$, a $\mathbb{Z}[t]$ -module	302
4.3.9	Other rings for other systems	303
4.3.10	The module-theoretic formulation of SE over a ring	304
4.3.11	Notes	305
4.4	Polynomial matrices	313
4.4.1	Background	313
4.4.2	Presenting SFTs with polynomial matrices	314
4.4.3	Algebraic invariants in the polynomial setting	316
4.4.4	Polynomial matrices: from elementary equivalence to conjugate SFTs	319

4.4.5	Classification of SFTs by positive equivalence in $I - \text{NZC}$	322
4.4.6	Functoriality: flow equivalence in the polynomial setting	324
4.4.7	Notes	326
4.5	Inverse problems for nonnegative matrices	330
4.5.1	The NIEP	330
4.5.2	Stable variants of the NIEP	331
4.5.3	Primitive matrices	331
4.5.4	Irreducible matrices	332
4.5.5	Nonnegative matrices	334
4.5.6	The Spectral Conjecture	335
4.5.7	Boyle–Handelman Theorem	337
4.5.8	The Kim–Ormes–Roush Theorem	338
4.5.9	Status of the Spectral Conjecture	338
4.5.10	Laffey’s Theorem	339
4.5.11	The Generalized Spectral Conjectures	340
4.5.12	Notes	342
4.6	A brief introduction to algebraic K-theory	346
4.6.1	K_1 of a ring $\mathcal{R}$	347
4.6.2	$NK_1(\mathcal{R})$	350
4.6.3	$\text{Nil}_0(\mathcal{R})$	352
4.6.4	K_2 of a ring $\mathcal{R}$	355
4.6.5	Notes	357
4.7	The algebraic K-theoretic characterization of the refinement of strong shift equivalence over a ring by shift equivalence	361
4.7.1	Comparing shift equivalence and strong shift equivalence over a ring	362
4.7.2	The Algebraic Shift Equivalence Problem	363
4.7.3	Strong shift equivalence and elementary equivalence	364
4.7.4	The refinement of shift equivalence over a ring by strong shift equivalence	366
4.7.5	The SE and SSE relations in the context of endomorphisms	369
4.7.6	Notes	370
4.8	Automorphisms of SFTs	373
4.8.1	Simple automorphisms	377
4.8.2	The center of $\text{Aut}(\sigma_A)$	379
4.8.3	Representations of $\text{Aut}(\sigma_A)$	380

4.8.4	Dimension representation	380
4.8.5	Periodic point representation	383
4.8.6	Inerts and the sign-gyration compatibility condition	385
4.8.7	Actions on finite subsystems	388
4.8.8	Notable problems regarding $\text{Aut}(\sigma_A)$	388
4.8.9	The stabilized automorphism group	390
4.8.10	Mapping class groups of subshifts	393
4.8.11	Notes	394
4.9	Wagoner's strong shift equivalence complex, and applications	399
4.9.1	Wagoner's SSE complexes	400
4.9.2	Homotopy groups for Wagoner's complexes and $\text{Aut}(\sigma_A)$	403
4.9.3	Counterexamples to Williams' conjecture	405
4.9.4	Kim–Roush relative sign-gyration method	407
4.9.5	Wagoner's K_2 -valued obstruction map	409
4.9.6	Some remarks and open problems	411
4.9.7	Notes	412
	<i>References</i>	414
	<i>Author Index</i>	423
	<i>Subject Index</i>	427