

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Higher Special Functions

A Theory of the
Central Two-Point Connection Problem
Based on a Singularity Approach

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CAMBRIDGE
UNIVERSITY PRESS

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