

# **Modern Quantum Theory**

## **From Quantum Mechanics to Entanglement and Quantum Information**

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