

GRADUATE STUDIES **230**
IN MATHEMATICS

Inverse Problems for Fractional Partial Differential Equations

Barbara Kaltenbacher
William Rundell

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