

Set Theory

An Introduction to Axiomatic Reasoning

Robert André



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

Contents

Preface	ix
I Axioms and classes	
1 Classes, sets and axioms	3
2 Constructing classes and sets	15
II Class operations	27
3 Operations on classes and sets	29
4 Cartesian products	38
III Relations	47
5 Relations on a class or set	49
6 Equivalence relations and order relations	55
7 Partitions induced by equivalence relations	66
8 Equivalence classes and quotient sets	72
IV Functions	79
9 Functions: A set-theoretic definition	81
10 Operations on functions	90
11 Images and preimages of sets	98
12 Equivalence relations induced by functions	104
V From sets to numbers	111
13 Natural numbers	113
14 The natural numbers as a well-ordered set	130
15 Arithmetic of the natural numbers	140
16 The integers $\mathbb{Z}$ and the rationals $\mathbb{Q}$	151
17 Real numbers: “Dedekind cuts are us!”	162
VI Infinite sets	173
18 Infinite sets versus finite sets	175
19 Countable and uncountable sets	188
20 Equipotence as an equivalence relation	198

21	The Schröder-Bernstein theorem	213
VII	Cardinal numbers	221
22	Introduction to cardinal numbers	223
23	Addition and multiplication in $\mathcal{C}$	233
24	Exponentiation of cardinal numbers	240
25	On sets of cardinality c	249
VIII	Ordinal numbers	261
26	More on well-ordered sets	263
27	Ordinals: definition and properties	279
28	Properties of the class of ordinals.	297
29	Cardinal numbers: “Initial ordinals are us!”	319
IX	Choice, regularity and Martin’s axiom	335
30	Axiom of choice	337
31	Axiom of regularity	352
32	Cumulative hierarchy	358
33	Martin’s axiom	374
X	Ordinal arithmetic	385
34	Ordinal addition	387
35	Ordinal multiplication and exponentiation	395
XI	Appendix	405
	Appendix A: Boolean algebras and Martin’s axiom	407
	Bibliography	423
	Index	425