

Functional Analysis

JAN VAN NEERVEN

Delft University of Technology



Contents

	<i>Preface</i>	<i>page</i> ix
	<i>Notation and Conventions</i>	xi
1	Banach Spaces	1
	1.1 Banach Spaces	2
	1.2 Bounded Operators	9
	1.3 Finite-Dimensional Spaces	18
	1.4 Compactness	21
	1.5 Integration in Banach Spaces	23
	Problems	28
2	The Classical Banach Spaces	33
	2.1 Sequence Spaces	33
	2.2 Spaces of Continuous Functions	34
	2.3 Spaces of Integrable Functions	47
	2.4 Spaces of Measures	64
	2.5 Banach Lattices	73
	Problems	77
3	Hilbert Spaces	87
	3.1 Hilbert Spaces	87
	3.2 Orthogonal Complements	92
	3.3 The Riesz Representation Theorem	95
	3.4 Orthonormal Systems	97
	3.5 Examples	100
	Problems	106
4	Duality	115
	4.1 Duals of the Classical Banach Spaces	115
	4.2 The Hahn–Banach Extension Theorem	128
	4.3 Adjoint Operators	137

4.4	The Hahn–Banach Separation Theorem	142
4.5	The Krein–Milman Theorem	144
4.6	The Weak and Weak* Topologies	147
4.7	The Banach–Alaoglu Theorem	152
	Problems	163
5	Bounded Operators	171
5.1	The Uniform Boundedness Theorem	171
5.2	The Open Mapping Theorem	174
5.3	The Closed Graph Theorem	176
5.4	The Closed Range Theorem	178
5.5	The Fourier Transform	180
5.6	The Hilbert Transform	190
5.7	Interpolation	192
	Problems	201
6	Spectral Theory	209
6.1	Spectrum and Resolvent	209
6.2	The holomorphic Functional Calculus	216
	Problems	224
7	Compact Operators	227
7.1	Compact Operators	227
7.2	The Riesz–Schauder Theorem	230
7.3	Fredholm Theory	234
	Problems	252
8	Bounded Operators on Hilbert Spaces	255
8.1	Selfadjoint, Unitary, and Normal Operators	255
8.2	The Continuous Functional Calculus	266
8.3	The Sz.-Nagy Dilation Theorem	272
	Problems	277
9	The Spectral Theorem for Bounded Normal Operators	281
9.1	The Spectral Theorem for Compact Normal Operators	281
9.2	Projection-Valued Measures	285
9.3	The Bounded Functional Calculus	287
9.4	The Spectral Theorem for Bounded Normal Operators	293
9.5	The Von Neumann Bicommutant Theorem	301
9.6	Application to Orthogonal Polynomials	305
	Problems	308
10	The Spectral Theorem for Unbounded Normal Operators	311
10.1	Unbounded Operators	311

10.2	Unbounded Selfadjoint Operators	322
10.3	Unbounded Normal Operators	326
10.4	The Spectral Theorem for Unbounded Normal Operators	332
	Problems	344
11	Boundary Value Problems	347
11.1	Sobolev Spaces	347
11.2	The Poisson Problem $-\Delta u = f$	372
11.3	The Lax–Milgram Theorem	385
	Problems	388
12	Forms	399
12.1	Forms	399
12.2	The Friedrichs Extension Theorem	410
12.3	The Dirichlet and Neumann Laplacians	411
12.4	The Poisson Problem Revisited	415
12.5	Weyl’s Theorem	416
	Problems	425
13	Semigroups of Linear Operators	427
13.1	C_0 -Semigroups	427
13.2	The Hille–Yosida Theorem	439
13.3	The Abstract Cauchy Problem	447
13.4	Analytic Semigroups	455
13.5	Stone’s Theorem	473
13.6	Examples	475
13.7	Semigroups Generated by Normal Operators	498
	Problems	500
14	Trace Class Operators	507
14.1	Hilbert–Schmidt Operators	507
14.2	Trace Class Operators	510
14.3	Trace Duality	521
14.4	The Partial Trace	523
14.5	Trace Formulas	526
	Problems	539
15	States and Observables	543
15.1	States and Observables in Classical Mechanics	543
15.2	States and Observables in Quantum Mechanics	545
15.3	Positive Operator-Valued Measures	561
15.4	Hidden Variables	574
15.5	Symmetries	577

15.6	Second Quantisation	598
	Problems	617
<i>Appendix A</i>	Zorn's Lemma	621
<i>Appendix B</i>	Tensor Products	623
<i>Appendix C</i>	Topological Spaces	627
<i>Appendix D</i>	Metric Spaces	637
<i>Appendix E</i>	Measure Spaces	647
<i>Appendix F</i>	Integration	661
<i>Appendix G</i>	Notes	671
	 <i>References</i>	 691
	<i>Index</i>	701