

Quantum Mechanics

A Mathematical Introduction

ANDREW J. LARKOSKI

SLAC National Accelerator Laboratory



CAMBRIDGE
UNIVERSITY PRESS

Contents

<i>Preface</i>	<i>page</i>	xi
Approach		xi
Structure of This Book		xii
Key Features		xiv
How to Teach With This Book		xiv
Acknowledgments		xv
1 Introduction		1
1.1 Structure of This Book		1
1.2 Fundamental Hypothesis of Quantum Mechanics		3
2 Linear Algebra		5
2.1 Invitation: The Derivative Operator		5
2.2 Linearity		6
2.3 Matrix Elements		8
2.4 Eigenvalues		12
2.5 Properties of the Derivative as a Linear Operator		16
2.6 Orthonormality of Complex-Valued Vectors		20
Exercises		22
3 Hilbert Space		27
3.1 The Hilbert Space		27
3.2 Unitary Operators		29
3.3 Hermitian Operators		33
3.4 Dirac Notation		36
3.5 Position Basis and Continuous-Dimensional Hilbert Spaces		40
3.6 The Time Derivative		44
3.7 The Born Rule		49
Exercises		54
4 Axioms of Quantum Mechanics and their Consequences		58
4.1 The Schrödinger Equation		58
4.2 Time Evolution of Expectation Values		63
4.3 The Uncertainty Principle		68
4.4 The Density Matrix: A First Look		80
Exercises		84

5	Quantum Mechanical Example: The Infinite Square Well	89
5.1	Hamiltonian of the Infinite Square Well	89
5.2	Correspondence with Classical Mechanics	98
	Exercises	103
6	Quantum Mechanical Example: The Harmonic Oscillator	109
6.1	Representations of the Harmonic Oscillator Hamiltonian	109
6.2	Energy Eigenvalues of the Harmonic Oscillator	112
6.3	Uncertainty and the Ground State	122
6.4	Coherent States	123
	Exercises	129
7	Quantum Mechanical Example: The Free Particle	134
7.1	Two Disturbing Properties of Momentum Eigenstates	134
7.2	Properties of the Physical Free Particle	136
7.3	Scattering Phenomena	139
7.4	The S-matrix	151
7.5	Three Properties of the S-matrix	154
	Exercises	165
8	Rotations in Three Dimensions	170
8.1	Review of One-Dimensional Transformations	170
8.2	Rotations in Two Dimensions in Quantum Mechanics	171
8.3	Rotations in Three Dimensions Warm-Up: Two Surprising Properties	175
8.4	Unitary Operators for Three-Dimensional Rotations	178
8.5	The Angular Momentum Hilbert Space	183
8.6	Specifying Representations: The Casimir of Rotations	187
8.7	Quantum Numbers and Conservation Laws	193
	Exercises	197
9	The Hydrogen Atom	201
9.1	The Hamiltonian of the Hydrogen Atom	201
9.2	The Ground State of Hydrogen	204
9.3	Generating All Bound States: The Laplace–Runge–Lenz Vector	210
9.4	Quantizing the Hydrogen Atom	214
9.5	Energy Eigenvalues with Spherical Data	223
9.6	Hydrogen in the Universe	229
	Exercises	237
10	Approximation Techniques	241
10.1	Quantum Mechanics Perturbation Theory	241
10.2	The Variational Method	247
10.3	The Power Method	251
10.4	The WKB Approximation	256
	Exercises	260

11	The Path Integral	268
11.1	Motivation and Interpretation of the Path Integral	268
11.2	Derivation of the Path Integral	271
11.3	Derivation of the Schrödinger Equation from the Path Integral	278
11.4	Calculating the Path Integral	282
11.5	Example: The Path Integral of the Harmonic Oscillator	285
	Exercises	294
12	The Density Matrix	299
12.1	Description of a Quantum Ensemble	299
12.2	Entropy	305
12.3	Some Properties of Entropy	307
12.4	Quantum Entanglement	314
12.5	Quantum Thermodynamics and the Partition Function	327
	Exercises	331
13	Why Quantum Mechanics?	337
13.1	The Hilbert Space as a Complex Vector Space	337
13.2	The Measurement Problem	339
13.3	Decoherence, or the Quantum-to-Classical Transition	341
A	Mathematics Review	345
B	Poisson Brackets in Classical Mechanics	351
C	Further Reading	354
	<i>Glossary</i>	356
	<i>Bibliography</i>	368
	<i>Index</i>	375