

Xavier Fernández-Real

Xavier Ros-Oton

Regularity Theory for Elliptic PDE



Contents

Preface	v
1 Overview and preliminaries	1
1.1 Preliminaries: Sobolev and Hölder spaces	3
1.2 A review on the Laplace equation	9
1.3 Probabilistic interpretation of harmonic functions	20
2 Linear elliptic PDE	25
2.1 Harnack's inequality	26
2.2 Schauder estimates for the Laplacian	34
2.3 Schauder estimates for operators in non-divergence form	47
2.4 Schauder estimates for operators in divergence form	60
2.5 The case of continuous coefficients	66
2.6 Boundary regularity	70
3 Nonlinear variational PDE and Hilbert's XIXth problem	73
3.1 Overview	74
3.2 Existence and basic estimates	77
3.3 De Giorgi's proof	83
3.4 Solution to Hilbert's XIXth problem	95
3.5 Further results and open problems	97
4 Fully nonlinear elliptic PDE	99
4.1 What is ellipticity?	99
4.2 Equations in two variables	104
4.3 Existence of solutions	107
4.4 Regularity of solutions: an overview	117
4.5 Further results and open problems	123
5 The obstacle problem	127
5.1 Some motivations and applications	130
5.2 Basic properties of solutions I	132
5.3 Basic properties of solutions II	142
5.4 Regularity of free boundaries: an overview	148
5.5 Classification of blow-ups	152
5.6 Regularity of the free boundary	163

5.7	Singular points	173
5.8	On the size of the singular set	177
A	Some properties of Hölder spaces	181
B	Proof of the boundary Harnack inequality	193
C	Probabilistic interpretation of fully nonlinear equations	203
D	Motivations and applications for the obstacle problem	211
	Notation	219
	References	221
	Index	227