

# **Complexity Science**

The Study of Emergence

**Henrik Jeldtoft Jensen**

Imperial College London



**CAMBRIDGE**  
UNIVERSITY PRESS

# Contents

Acknowledgements	<i>page xi</i>
Nomenclature	xiii
Preface	xvii
<b>I Conceptual Foundation of Complexity Science</b>	<b>1</b>
<b>Introduction to Part I</b>	<b>3</b>
<b>1 The Science of Emergence</b>	<b>5</b>
1.1 The Importance of Interaction	9
1.2 Past Views on Emergence	15
1.3 Further Reading	18
1.4 Exercises and Projects	19
<b>2 Conceptual Framework of Emergence</b>	<b>21</b>
2.1 Emergence of a Characteristic Scale or Lack of Scale	23
2.2 Emergence of Collective Robust Degrees of Freedom	26
2.3 Structural Coherence	28
2.4 Evolutionary Diffusion	31
2.5 Breaking of Symmetry	33
2.6 Emergence of Networks	35
2.7 Temporal Mode	37
2.8 Adaptive and Evolutionary Dynamics	39
2.9 Further Reading	40
2.10 Exercises and Projects	41
<b>3 Specific Types of Emergent Behaviour</b>	<b>46</b>
3.1 Ising-Type Models: Transitions and Criticality	48
3.2 Network Models and Scale vs. No Scale	52
3.3 Emergence of Coherence in Time: Synchronisation	57
3.4 Evolutionary Dynamics: Adaptation	60
3.5 Mean-Field Modelling: Dimensionality and Forecasting	64
3.6 Further Reading	69
3.7 Exercises and Projects	70

<b>4</b>	<b>The Value of Prototypical Models of Emergence</b>	<b>75</b>
4.1	The Need for Simplification of Models	76
4.2	O’Keeffe–Einstein Propositions at Work	78
4.3	Further Reading	82
4.4	Exercises and Projects	83
<b>II</b>	<b>Mathematical Tools of Complexity Science</b>	<b>87</b>
	<b>Introduction to Part II</b>	<b>89</b>
<b>5</b>	<b>Branching Processes</b>	<b>93</b>
5.1	Generator Functions: Sizes and Lifetimes	97
5.1.1	Size of the Progeny	99
5.1.2	Time to Extinction	102
5.2	Branching Trees and Random Walks	103
5.3	Further Reading	106
5.4	Exercises and Projects	107
<b>6</b>	<b>Statistical Mechanics</b>	<b>110</b>
6.1	Probabilities and Ensembles	110
6.2	The Ising Model	119
6.3	The Peculiar Nature of the Critical Point	125
6.4	Fluctuations, Response and Correlations	127
6.5	Examples of Correlation Functions: Brain, Flocks of Birds, Finance	132
6.6	Diverging Range of Correlations	133
6.6.1	Correlation Function – Exact Approach	134
6.6.2	Correlation Function – Intuitive Discussion	139
6.7	The Two-Dimensional XY Model	143
6.7.1	2d XY: Some Mathematical Details	148
6.7.2	Vortex Unbinding	153
6.7.3	The Vortex Unbinding Transition in Other Systems	154
6.8	Further Reading	156
6.9	Exercises and Projects	156
<b>7</b>	<b>Synchronisation</b>	<b>163</b>
7.1	The Kuramoto Model: The Onset of Synchronisation	164
7.2	Chimera States	170
7.3	Further Reading	174
7.4	Exercises and Projects	175
<b>8</b>	<b>Network Theory</b>	<b>177</b>
8.1	Basic Concepts	178
8.2	Measures of the Importance of Nodes	179

8.2.1	Degree Centrality	179
8.2.2	Eigenvector Centrality	184
8.2.3	Closeness Centrality	187
8.2.4	Betweenness Centrality	187
8.2.5	How Well Does it Work?	188
8.3	Community Detection	188
8.4	Spreading on Networks – Giant Cluster	196
8.5	Analysis of Dynamics of and on Networks	203
8.5.1	Generating Networks	204
8.5.2	Random Walk on Networks	212
8.5.3	Synchronisation on Networks	216
8.6	Further Reading	224
8.7	Exercises and Projects	225
<b>9</b>	<b>Information Theory and Entropy</b>	<b>230</b>
9.1	Information Theory and Interdependence	232
9.2	Entropy and Estimates of Causal Relations	237
9.3	From Time Series to Networks	241
9.4	From Entropy to Probability Distribution	245
9.5	Measures of Degrees of Complexity	256
9.5.1	Lempel–Ziv Complexity Measure	256
9.5.2	Information-Theoretic Approach to Emergence	259
9.5.3	Group Entropy Measure of Complexity	272
9.6	Further Reading	274
9.7	Exercises and Projects	275
<b>10</b>	<b>Stochastic Dynamics and Equations for the Probabilities</b>	<b>279</b>
10.1	Random Walk and Diffusion	280
10.2	First Passage and First Return Times	293
10.3	Correlations in Time	297
10.4	Random Walk with Persistence or Anti-persistence: Hurst Exponent	302
10.5	Stationary Diffusion: Ornstein–Uhlenbeck Process	307
10.6	Evolutionary Dynamics and Clustering	309
10.7	Master Equation, Coarse Graining and Free Energy	313
10.8	Further Reading	318
10.9	Exercises and Projects	319
<b>11</b>	<b>Agent-Based Modelling</b>	<b>324</b>
11.1	Flocks of Birds or Schools of Fish	325
11.2	Models of Segregation	328
11.3	The Tangled Nature Model	337
11.4	Further Reading	349
11.5	Exercises and Projects	350

<b>12 Intermittency</b>	<b>356</b>
12.1 Self-Organised Criticality	357
12.1.1 Sandpile Models	358
12.1.2 Mean-Field Analysis	361
12.1.3 Lessons from Sandpile Models	364
12.1.4 Forest Fire Model	367
12.2 Record Dynamics	370
12.2.1 Statistics of Records	371
12.2.2 Spin Glasses, Superconductors, Ants and Evolution	375
12.3 Tangent Map Intermittency	379
12.4 Further Reading	382
12.5 Exercises and Projects	383
 <b>13 Tipping Points, Transitions and Forecasting</b>	 <b>387</b>
13.1 Externally Induced Transitions	387
13.2 Intrinsic Instability	389
13.3 Further Reading	395
13.4 Exercises and Projects	395
 <b>14 Concluding Comments and a Look to the Future</b>	 <b>397</b>
14.1 Further Reading	399
 Glossary	 401
References	411
Index	436